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HW1 , Math 531, Spring 2014

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QUESTION 1. (i) Let R be a ring with one. Prove that $-1.a = -a$ for every $a \in R$ (i.e., the additive inverse of 1 times a equals to the additive inverse of a.)

(ii) Let $R = Z \times Z$. For $(a, b), (c, d) \in R$, define (a, b) + (c, d) = (a + c, b + d) and (a, b).(c, d) = (ac + ad + bc, bd). Then R is a commutative ring with identity (DO NOT SHOW THAT). Show the following

a. what is the identity of R?

b. Find all units of R

(iii) Let $R = Z_9(+)Z_9$. For $(a, b), (c, d) \in R$, define $(a, b) + (c, d) = (a + c, b + d)$ and $(a, b).(c, d) = (ac, bc + ad)$. Then R is a commutative ring with 1 (Do not show that). Show the following

a. what is the identity of R?

b. Find all units of R.

(iv) Let $R = \{\begin{bmatrix} a & b \\ -b & a \end{bmatrix} |a, b \in \mathbb{Q}\}$. Show that $(R, +, .)$ is a field.

(v) Let $R = \{\begin{bmatrix} a & b \\ -b & a \end{bmatrix} |a, b \in \mathbb{Z}\}$. Show that $(R, +, .)$ is never a field. Find $U(R)$.

(vi) Let $R = \{\begin{bmatrix} a+bi & c+di \\ -c+di & a-bi \end{bmatrix} |a, b, c, d \in \mathbb{Q}\}$. Show that $(R, +, .)$ is a division ring but never a field (note that $i^2 = -1$).

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